Module 3: Investigating Growth and Decay

TOPIC 2: USING EXPONENTIAL EQUATIONS

In this topic, students explore strategies for distinguishing exponential functions that represent growth scenarios versus those that represent decay, and methods for solving exponential equations. Students begin by comparing the value of a simple interest account and a compound interest account. They graph and write equations for these two scenarios and then compare the average rate of change of each for a given interval. Students then examine the structure of the exponential equations to recognize scenarios in which exponential functions grow or decay by a certain percent. Throughout the rest of the topic, students solve real-world problems that can be modeled by exponential functions, including one that requires students to combine function types to best model the scenario.

Where have we been?

Students know the rules of exponents and are familiar with the structure of exponential functions from their work in the previous topic. Their previous work to transform an exponential function has prepared them to make sense of real-world scenarios that they are modeling in this topic. Students understand what it means to determine a solution for an equation.

Where are we going?

This topic represents students' first deep dive into solving equations that represent nonlinear functions. As students gain proficiency in solving increasingly complex equations, they are able to model more interesting and complex real-life phenomena.

Exponential Growth and Decay

An exponential growth function has a b-value greater than 1 and is of the form $y = a \cdot (1 + r)^x$, where r is the rate of growth. The b-value is $1 + r$. An exponential decay function has a b-value greater than 0 and less than 1 and is of the form $y = a \cdot (1 - r)^x$, where r is the rate of decay. The b-value is $1 - r$.

I Feel the Earth. Move.

How do scientists measure the intensity of earthquakes? You may know that scientists who study earthquakes—seismologists—refer to a scale known as a Richter scale when reporting the strength of an earthquake. The Richter scale is a kind of exponential scale.

The scale generally goes from 1 to 9 (though it doesn't really have an upper limit), but an earthquake which has an intensity of 6 on the Richter scale is 10 times more powerful than an earthquake which measures 5.

One of the strongest earthquakes in history occurred in Chile on May 22, 1960. This earthquake measured an amazing 9.5 on the Richter scale—over 30,000 times stronger than a magnitude 5 earthquake!

Talking Points

Exponential functions in real-world contexts is an important topic to know about for college admissions tests.

Here is a sample question:

A car valued at $21,000 depreciates at a rate of 17% per year. What is the value of the car after 5 years?

To solve this, students should know to use the model for exponential decay, $y = a \cdot (1 - r)^x$, where a represents the initial value, r represents the rate of decrease, and x represents time.

$$y = a(1 - r)^x$$
$$y = a(1 - 0.17)^x$$
$$y = 21{,}000(0.83)^5$$
$$y = 8271.99$$

In 5 years, the car will be worth $8271.99.

Key Terms

simple interest

In a simple interest account, a percent of the starting balance is added to the account at each interval. The formula for simple interest is $I = Prt$, where P represents the starting amount, or principal, r represents the interest rate, t represents time, and I represents the interest earned.

compound interest

In a compound interest account, the balance is multiplied by the same amount at each interval.